Midterm Exam

Statistics 1XXX

1. *Descriptive statistics: mean, median, standard deviation, IQR, outliers etc.*

2. *Probability rules, conditional probability, independence etc.* $P(A) = .4$, $P(B) = .3$, and $P(A \text{ or } B) = .58$ Are A and B independent? Mutually exclusive?

   Solution.

   The events A and B are independent if and only if

   $$P(A \text{ and } B) = P(A)P(B).$$

   We know $P(A)$ and $P(B)$, so let us find $P(A \text{ and } B)$. By addition rule, we have

   $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

   So if $P(A \text{ and } B) = x$ we obtain the following equation:

   $$.58 = .4 + .3 - x,$$

   therefore, $x = .12$. Since $.4 \times .3 = .12$ we obtain that

   $$P(A \text{ and } B) = P(A)P(B),$$

   that is, A and B are independent. Since $P(A \text{ and } B) = .12 \neq 0$, events A and B are not mutually exclusive.

3. *Probability table.* An investment firm has classified its clients according to their gender and the composition of their investment portfolio (primarily bonds, primarily stocks, or a balanced mix of bonds and stocks). The numbers of clients falling into the various categories are shown in the following table:

	Portfolio Composition		
Gender	Bonds	Stocks	Balanced
Male	180	200	250
Female	120	100	150

Find the following probabilities:

a. The probability that the client selected either is male or has a balanced portfolio or both.

b. The probability that the client selected is female and has a bonds portfolio.

c. The probability that the employee selected is male, given that the employee has an unbalanced portfolio.

d. The probability that the employee selected is female, given that the employee has a balanced portfolio.

e. Two events A and B are defined as follows:

A: The client selected is male.

B: The client selected has a balanced portfolio.

Are A and B independent events? Explain.

Solution.

a. $P(\text{male or balance}) = (180 + 200 + 250 + 150)/1000 = 780/1000$.

b. $P(\text{female and bonds}) = 120/1000$.

c. This is the conditional probability:

$$\begin{aligned} P(\text{male} \mid \text{unbalanced}) &= \frac{P(\text{male and unbalanced})}{P(\text{unbalanced})} \\ &= \frac{(180 + 200)/1000}{(180 + 200 + 120 + 100)/1000} \\ &= \frac{380}{600}. \end{aligned}$$

d.

$$\begin{aligned} P(\text{female} \mid \text{balanced}) &= \frac{P(\text{female and balanced})}{P(\text{balanced})} \\ &= \frac{150}{400}. \end{aligned}$$

e. From the table we find first that

$$P(A) = 630/1000, \quad P(B) = 400/1000, \quad \text{and} \quad P(A \text{ and } B) = 250/1000.$$

Since $P(A \text{ and } B) \neq P(A)P(B)$, events A and B are NOT independent.

4. *Probability Tree.* At the beginning of each year, an investment newsletter predicts whether or not the stock market will rise over the coming year. Historical evidence reveals that there is a 75% chance that the stock market will rise in any given year. The newsletter has predicted a rise for 80% of the years when the market actually rose, and has predicted a rise for 40% of the years when the market fell.
a. Find the probability that the newsletters prediction for next year will be correct.
b. Find the probability that the market actually will rise if the newsletter has predicted a rise.

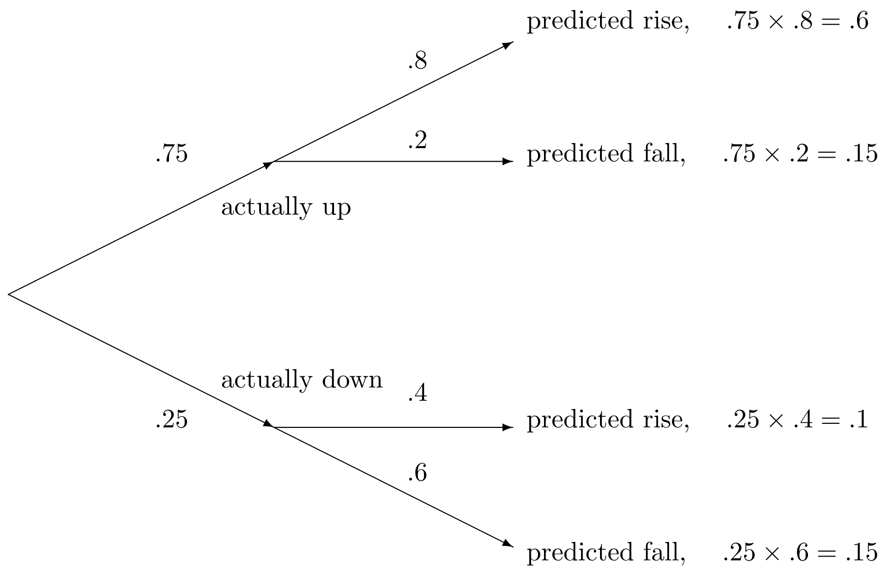

Figure 1: Tree Diagram. Investment Newsletter Problem

Solution.

a. $P(\text{correct}) = P(\text{up and rise}) + P(\text{down and fall}) = .6 + .15 = .75$

b. This is the conditional probability:

$$P(\text{actually up} \mid \text{predicted rise}) = \frac{P(\text{up and rise})}{P(\text{up and rise}) + P(\text{down and rise})} = \frac{.6}{.6 + .1} = \frac{6}{7}.$$

5. *Binomial distribution.* An official from the securities commission estimates that 70% of all investment bankers have profited from the use of insider information. 15 investment bankers are selected at random from the commissions registry.

a. Find the probability that all 15 have profited from insider information.

b. Find the probability that at least 14 have profited from insider information.

c. What is the expected number of bankers that have profited from insider information

Solution.

Let X be the number of bankers in the sample who have profited from the use of insider information. Then $X \sim b(15, .7)$.

a. By the binomial formula: $P(X = 15) = C_{15}^{15} \times .7^{15} \times (1 - .7)^{(15-15)} = 1 \times .7^{15} \times 1 = .7^{15} \approx .005$

b. Using the table:

$$\begin{aligned} P(X \geq 14) &= P(X = 14) + P(X = 15) \\ &= .031 + .005 \\ &= .036 \end{aligned}$$

c. By formula, $E(X) = np = 15 \times .7 = 10.5$.