1. Suppose that $X_1, ..., X_{n_1}$ and $Y_1, ..., Y_{n_2}$ constitute independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Then the Central Limit Theorem can be extended to show that $\bar{X} - \bar{Y}$ is approximately normally distributed for large n_1 and n_2 with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2/n_1 + \sigma_2^2/n_2$.

   Water flow through soils depends, among other things, on the porosity (volume proportion due to voids) of the soil. To compare two types of sandy soil, $n_1 = 50$ measurements are to be taken on the porosity of soil A, and $n_2 = 100$ measurements are to be taken on soil B. Assume that $\sigma_1^2 = 0.01$ and $\sigma_2^2 = 0.02$. Find the approximate probability that the difference between the sample means will be within 0.05 unit of the true difference between the population means, $\mu_1 - \mu_2$.

   Solution:
   Since $\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_1 - \mu_2, .0004)$ we get

$$\begin{aligned} P(|\bar{X} - \bar{Y} - (\mu_1 - \mu_2)| < .05) &= P\left(|Z| < \frac{.05}{\sqrt{.0004}}\right) \\ &= P(|Z| < 2.5) = .9876. \end{aligned}$$

2. Refer to previous exercise. Suppose samples are to be selected with $n_1 = n_2 = n$. Find the value of n that will allow the difference between the sample means to be within 0.04 unit of $\mu_1 - \mu_2$ with probability approximately 0.90.

   Solution:
   In this case $\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_1 - \mu_2, .03/n)$. We need to find n such that

$$P(|\bar{X} - \bar{Y} - (\mu_1 - \mu_2)| < .04) = .9,$$

   or

$$P(|Z| < .04/\sqrt{.03/n}) = .9.$$

   That is,

$$.04/\sqrt{.03/n} = 1.645.$$

   and

$$n \approx 51.$$