Review **Stat 3445**
Final Exam **May 3, 1-3pm, CLAS 313**

1. Let $X_1, X_2, ..., X_n$ be a random sample from the following distribution:

$$f(x) = \begin{cases} \frac{2x}{\theta} e^{-x^2/\theta}, & x \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find a sufficient statistic for θ.

2. Let $X_1, X_2, ..., X_n$ be a random sample from the following distribution:

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find a sufficient statistic for θ.

3. Let $X_1, X_2, ..., X_n$ be a random sample from the following distribution:

$$f(x) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find a sufficient statistic for θ.

4. Let $X_1, X_2, ..., X_n$ be a random sample from the following distribution:

$$f(x) = \begin{cases} \frac{1}{\theta} x^{\frac{1}{\theta}-1}, & 0 \leq x \leq 1,\ \theta > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Let $Y_i = -\ln(X_i)$. Show that $\sum_{i=1}^n Y_i$ is sufficient for θ.
   (b) Calculate $P(Y_i > y)$. What is the distribution of Y_i? What is $E(Y_i)$?
   (c) What is the MVUE for θ?

5. Let $X_1, X_2, ..., X_n$ be a random sample from the following distribution:

$$f(x) = \begin{cases} \alpha x^{\alpha-1}, & 0 \leq x \leq 1,\ \alpha > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) What is the MLE for α?
   (b) What is the MLE for $1/\alpha$?

6. The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of an alloy of steel, the Rock well hardness index averaged 62 with standard deviation 8. The manufacturer claims that this alloy has an average hardness index of at least 64.

   (a) Is there sufficient evidence to refute the manufacturer's claim at the 1% significance level?

   (b) Calculate the value of β for the alternative $\mu_a = 60$.

7. A precision instrument is guaranteed (by a manufacturer) to read accurately within standard deviation of 2 units. A sample of four instrument readings on the same object yielded the measurements 353, 351, 351, and 355. Are these readings are more variable than what is guaranteed by the manufacturer? Use $\alpha = .01$.

8. A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790, 793, and 802 tons. Assume that the daily yields are normally distributed.

   (a) Do these data indicate that the mean daily yield is below 800 tons? Use $\alpha = .05$.

   (b) Bound the p-value using the t-distribution table.

9. A small amount of the trace element selenium, from 50 to 200 micrograms (mg) per day, is considered essential to good health. Suppose that independent random samples of $n_1 = n_2 = 50$ adults were selected from two regions of the United States, and a day's intake of selenium, from both liquids and solids, was recorded for each person. The mean and standard deviation of the selenium daily intakes for the 50 adults from region 1 were $\bar{X}_1 = 170$ mg and $s_1 = 25$ mg, respectively. The corresponding statistics for the 50 adults from region 2 were $\bar{X}_2 = 140$ mg and $s_2 = 15$ mg.

   (a) Is the difference in the selenium daily intakes from the two regions statistically significant? Use $\alpha = .05$.

   (b) Find the p-value.