Statistics 3515

Lecture 3

Tests on Means Set Prior to Experimentation in One Way ANOVA

a. Orthogonal Contrasts

A ***contrast*** in treatment means is a linear combinations of the form

$$\Gamma = \sum_{i=1}^{a} c_i \mu_i,$$

where

$$\sum_{i=1}^{a} c_i = 0.$$

We say that a linear combination of treatment averages, C, is a ***contrast*** corresponding to the contrast Γ if

$$C = \sum_{i=1}^{a} c_i \bar{y}_i \qquad (1)$$

and

$$\sum_{i=1}^{a} n_i c_i = 0. \qquad (2)$$

If the design is balanced then (1) is a contrast if

$$\sum_{i=1}^{a} c_i = 0. \qquad (3)$$

Two contrasts C_1 and C_2 are orthogonal to each other if

$$\sum_{i=1}^{a} n_i c_{i1} c_{i2} = 0, \qquad (4)$$

and if the design is balanced then condition (4) is replaced by

$$\sum_{i=1}^{a} c_{i1} c_{i2} = 0. \qquad (5)$$

The sum of squares for any contrast C is given by

$$SS_C = \frac{C^2}{\sum_{i=1}^{a} c_i^2/n_i} .$$

and it has one degree of freedom. For a treatments the set of a - 1 orthogonal contrasts partition the sum of squares for treatment into a - 1 independent singe-degree-of-freedom components.

To test H_0: $\Gamma = 0$ vs H_a: $\Gamma \neq 0$ the following test statistics is used

$$F = \frac{SS_C}{MS_E},$$

which has an F distribution with 1, N–a degrees of freedom (N is the total number of observations in the experiment).

The a - 1 tests of the corresponding linear combination means are independent of each other.

Usually, something in the nature of the experiment should suggest which comparisons will be of interest. Contrast coefficients must be chosen prior to running the experiment and examining the data. The reason for that is, if comparisons are selected after examining the data, most experimenters would construct tests that correspond to large observed differences in means. These large differences could be the result of real effects or they could be the result of random error. If experimenters consistently pick the largest differences to compare, they will inflate the type I error of the test since it is likely that, in an unusually high percentage of comparisons selected, the observed differences will be the result of random error.

b. Comparing Treatments with a Control.

In some experiments one of the treatments (say a) is the standard treatment, *a control*, and the experimenter wants to compare it with a-1 new treatments. In this case one is interested in a-1 comparisons. A procedure to perform these comparisons has been studied by Dunnett (1964), which is a modification of the t-test. We are interested in testing:

$$H_0\text{: } \mu_i = \mu_a \quad \text{vs} \quad H_0\text{: } \mu_i \neq \mu_a$$

where i=1,....,a-1. The null hypotheses H_0: $\mu_i = \mu_a$ is rejected at the α level if

$$|\bar{y}_{i.} - \bar{y}_{a.}| > d_\alpha(a - 1, f)\ [MS_E\ (1/n_i + 1/n_a)]^{.5}$$

where the constant $d_\alpha(a - 1, f)$ is given in Table VI on page A-13. One sided tests are also possible (page A-14). This test controls the overall significance level.

Remark: It is advisable to use more observations for the control group. The ratio $n_a / n_i \approx \sqrt{a}$ is recommended.