R code for section 2.3 of "Threshold modeling of nonstationary extremes"

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1 Required software

R is a free software environment for statistical computing and graphics. It can be installed from `www.http://cran.r-project.org/`. Our code uses the R package quantreg (Koenker, 2011) `http://cran.r-project.org/web/packages/quantreg/index.html`. The R commands used below are given in the file `NJR.R`.

2 Getting started

The R package quantreg must be installed first. Then we (a) load the quantreg package; (b) set the working directory to one containing the files `NJR.fns` and the data files `NJRWAMdata.txt` and `NJRdailyNAO.txt`; (c) read in the functions contained in `NJR.fns`.

```
> # load quantreg
> library(quantreg)
> # Set work directory
> setwd("C:/Users/paul/Documents/EV_BOOK/R")
> # input functions to fit NHPP regression models etc
> source("NJR.fns")
```

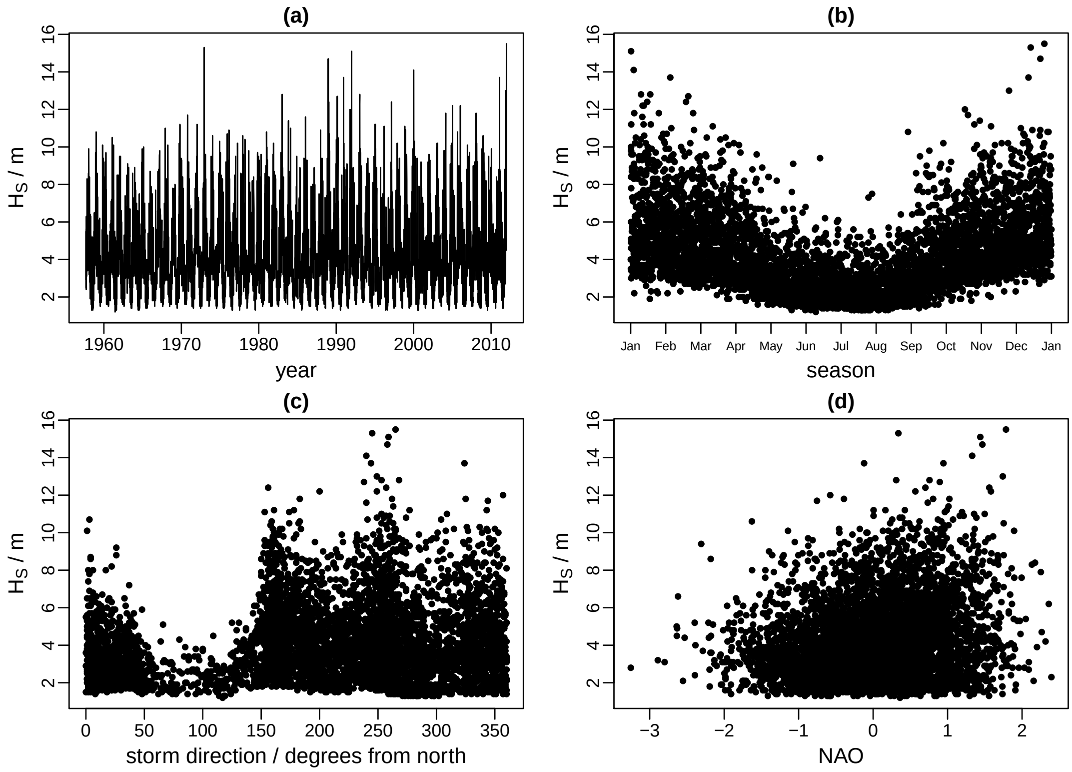
3 Data

We read in the WAM hindcast data and the daily NAO data and merge them to form a single dataframe.

```
> # Read WAM hindcast data ...
> my.names <- c("Hs","direction","season","peak.date","start.date","end.date")
> WAM <- read.table("NJRWAMdata.txt",col.names=my.names,colClasses="numeric")
> # Create dates, bearing in mind the Matlab time origin
> HsMaxDates <- as.Date(WAM[,"peak.date"]-1, origin="0000-01-01")
> POT.year <- as.numeric(substr(HsMaxDates,1,4))  # year of HS peak
> POT.month <- as.numeric(substr(HsMaxDates,6,7))  # month of HS peak
> POT.day <- as.numeric(substr(HsMaxDates,9,10))  # year of HS peak
> # Create dataframe ...
> POT <- cbind(Hs=WAM$Hs,direction=WAM$direction,season=WAM$season/360,
+              year=POT.year,month=POT.month,day=POT.day)
> # Read NOAA daily NAO data ...
> NAO.daily <- read.table("NJRdailyNAO.txt",header=F,fill=T,col.names=c("year","month","day","NAO"))
> # interpolate linearly to avoid 2 missing NAO values (26/10/2006, 26/1/2007)
> which.na <- which(is.na(NAO.daily[,"NAO"]))[1:2]  # Find the (two) missing value in 1957-2011
> NAO.daily[which.na,"NAO"] <- (NAO.daily[which.na-1,"NAO"]+NAO.daily[which.na+1,"NAO"])/2
> # Add to hindcasts dataframe ...
> POT <- merge(POT,NAO.daily,by=c("year","month","day"),sort=FALSE)
> attach(POT) # attach dataframe, so that its variables are directly accessible
```

3.1 Exploratory plot

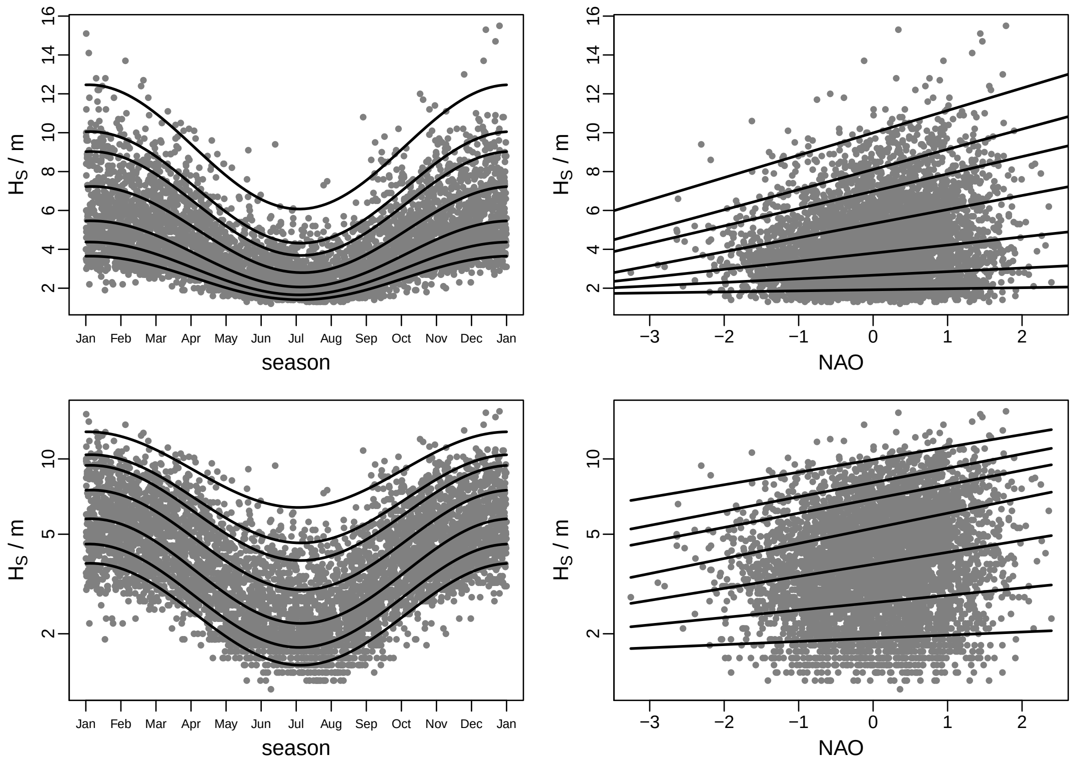
```
> par(mfrow=c(2,2),mar=c(3,3.5,1.5,0.75),lwd=1,cex.lab=1.25,cex.axis=1,mgp=c(3,0.5,0))
> my.pch <- 20; my.line <- 1.75
> my.ylab <- expression(paste(H[S]," / m"))
> plot(HsMaxDates,Hs,type="l",ann=F); title(xlab="year",ylab=my.ylab,line=my.line)
> title(main="(a)",line=0.5)
> plot(season,Hs,ann=F,pch=my.pch,axes=F); title(xlab="season",ylab=my.ylab,line=my.line)
> title(main="(b)",line=0.5)
> axis(1,at=seq(from=0,by=1/12,len=13),labels=c(month.abb,month.abb[1]),cex.axis=0.75)
> axis(2); box()
> plot(direction,Hs,ann=F,pch=my.pch)
> title(xlab="storm direction / degrees from north",ylab=my.ylab,line=my.line)
> title(main="(c)",line=0.5)
> plot(NAO,Hs,ann=F,pch=my.pch); title(xlab="NAO",ylab=my.ylab,line=my.line)
> title(main="(d)",line=0.5)
```



3.2 Quantile regression

```
> my.probs <- c(0.1,0.25,0.5,0.75,0.9,0.95,0.99) # some conditional quantile levels
> #-------------- Hs scale (additive effects)
> # NAO
> rq.NAO <- rq(Hs~NAO,tau=my.probs) # quantile regression: Hs vs. NAO
> #
> # seasonal degree
> cos.1 <- cos(2*pi*season) # period 1 year (360 days)
> sin.1 <- sin(2*pi*season)
> #
> rq.season.1 <- rq(Hs~cos.1+sin.1,tau=my.probs) # quantile regression: Hs vs. sine wave
> #-------------- log(Hs) scale (multiplicative effects)
> # NAO, log(Hs)
> rq.NAO.log <- rq(log(Hs)~NAO,tau=my.probs) # quantile regression: Hs vs. NAO
> # seasonal degree, log(Hs)
> rq.season.1.log <- rq(log(Hs)~cos.1+sin.1,tau=my.probs) # quantile regression: Hs vs. sine wave
```

```
>  par(mfrow=c(2,2),mar=c(3,3.5,1,0.75),lwd=1,cex.lab=1.25,cex.axis=1,mgp=c(3,0.5,0))
>  my.pch <- 20; my.line <- 1.75;  my.grey <- grey(0.5); my.ylab <- expression(paste(H[S]," / m"))
> # Hs : seasonal degree
>  plot(season,Hs,ann=F,pch=my.pch,col=my.grey,axes=F)
>  title(xlab="season",ylab=my.ylab,line=my.line)
>  axis(1,at=seq(from=0,by=1/12,len=13),labels=c(month.abb,month.abb[1]),cex.axis=0.75); axis(2); box()
>  xx <- seq(0,1,len=101); xx.sin <- sin(2*pi*xx); xx.cos <- cos(2*pi*xx)
>  yy <- cbind(1,xx.cos,xx.sin)%*%rq.season.1$coeff
>  for (i in 1:length(my.probs)) lines(xx,yy[,i],lwd=2)
> # Hs : NAO
>  plot(NAO,Hs,ann=F,pch=my.pch,col=my.grey); title(xlab="NAO",ylab=my.ylab,line=my.line)
>  for (i in 1:length(my.probs)) abline(coef=rq.NAO$coeff[,i],lwd=2)
> # log(Hs) : seasonal degree
>  plot(season,Hs,ann=F,pch=my.pch,col=my.grey,axes=F,log="y")
>  title(xlab="season",ylab=my.ylab,line=my.line)
>  axis(1,at=seq(from=0,by=1/12,len=13),labels=c(month.abb,month.abb[1]),cex.axis=0.75); axis(2); box()
>  xx <- seq(0,1,len=101); xx.sin <- sin(2*pi*xx); xx.cos <- cos(2*pi*xx)
>  yy <- cbind(1,xx.cos,xx.sin)%*%rq.season.1.log$coeff
>  for (i in 1:length(my.probs)) lines(xx,exp(yy[,i]),lwd=2)
> # log(Hs) : NAO
>  plot(NAO,Hs,ann=F,pch=my.pch,col=my.grey,log="y")
>  title(xlab="NAO",ylab=my.ylab,line=my.line)
>  xx.NAO <- seq(min(NAO),max(NAO),len=101)
>  yy <- cbind(1,xx.NAO)%*%rq.NAO.log$coeff
>  for (i in 1:length(my.probs)) lines(xx.NAO,exp(yy[,i]),lwd=2)
```



4 NHPP regression modelling

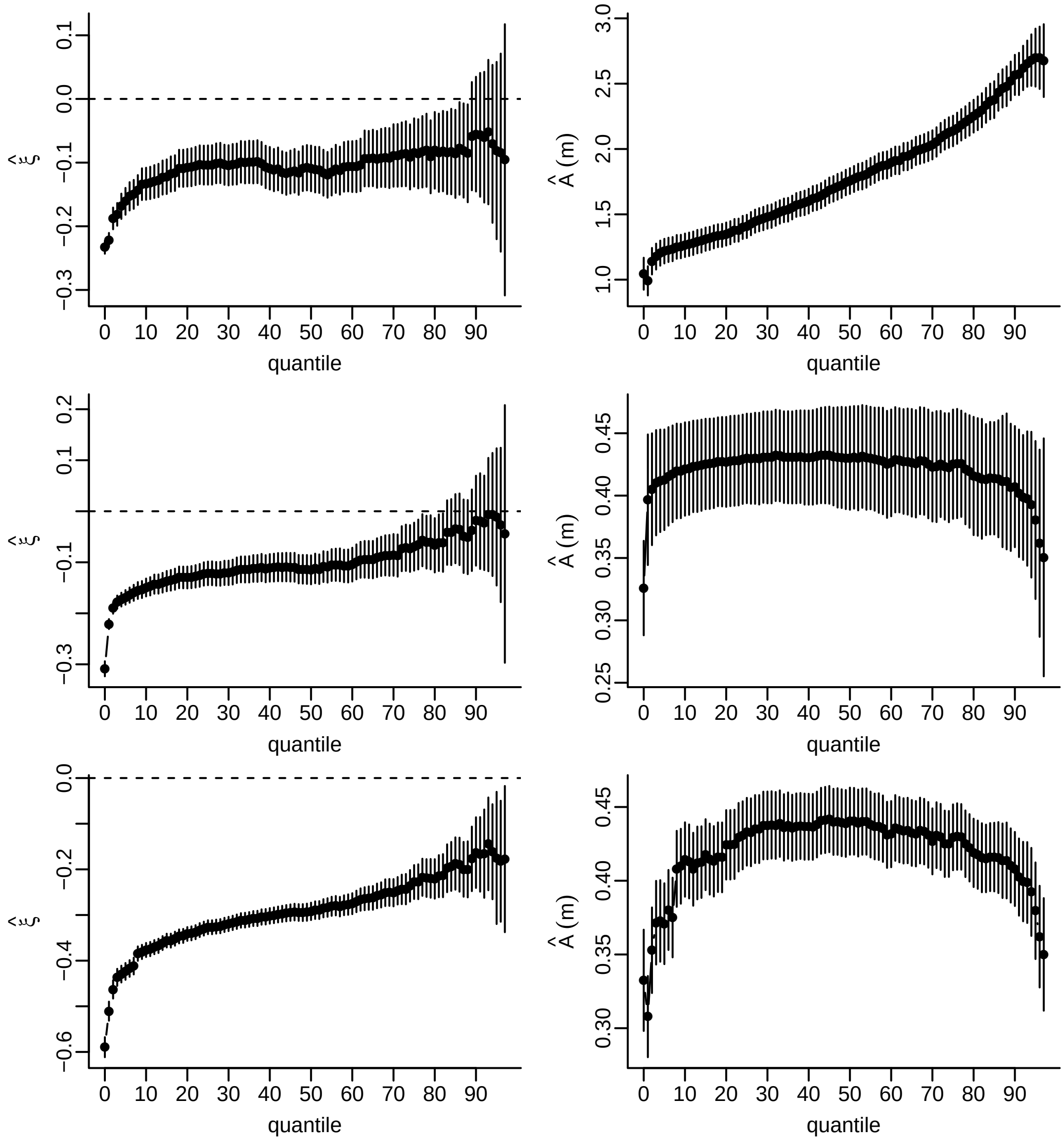
```
> # Create a covariate matrix: with columns
> # 1: NAO
> # 2: cos.1 - cosine with period 360 days
> # 3: sin.1 - sine with period 360 days
> # 4: NAO*cos.1 interaction
> # 5: NAO*sin.1 interaction
>
> xdat <- cbind(NAO,cos.1,sin.1,NAO.cos.1=NAO*cos.1,NAO.sin.1=NAO*sin.1)
```

4.1 Threshold estimation using quantile regression

```
> # Each of the following commands involves multiple model fits and will
> # therefore take a few minutes to run
>
> pmin <- 0; pmax <- 97 # min and max conditional quantile levels
> nint <- (pmax-pmin)+1  # number of threshold levels
>
> # Model 1: response Hs, mu linear in covariates
> #model.1 <- NHPP.fitrange.cov(Hs,xdat=xdat,mul=1:5,pmin=pmin,pmax=pmax,nB=1,nint=nint)
>
> # NHPP.fitrange.cov() vs NHPP.fitrange.cov.new()? Need use.rq=T?
>
> # Model 2: response Hs, log(mu) linear in covariates, sigma proportional to mu
> #model.2 <- NHPP.fitrange.cov(Hs,xdat=xdat,mul=1:5,pmin=pmin,pmax=pmax,nB=1,nint=nint,
> #                              mulink=exp,sig.propto.m=T)
>
> # Model 3: response log(Hs), mu linear in covariates
> #model.3 <- NHPP.fitrange.cov(log(Hs),xdat=xdat,mul=1:5,pmin=pmin,pmax=pmax,nB=1,nint=nint)
```

The file `NJR.R` contains code to plot all parameters against threshold for each of the models.

```
> which.ps <- which(0:97 %in% c(0,10,20,30,40,50,60,70,80,90))
> my.ylab <- c(expression(hat(mu)[0] ~(m)),expression(hat(alpha)[N] ~(m)),expression(hat(A) ~(m)),
+    expression(hat(phi) ~(days)),expression(hat(A)[N] ~(m)),expression(hat(phi)[N] ~(days)),
+    expression(hat(sigma) ~(m)),expression(hat(xi)))
>  par(mfrow=c(3,2))
>  NHPP.fitrange.plot(model.1,ngraphs=2,par.order=c(8,3),ncolumns=2,my.ylab=my.ylab,
+    which.ps=which.ps,cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T),my.cex.lab=1,set.cols=F)
>  NHPP.fitrange.plot(model.2,ngraphs=2,par.order=c(8,3),ncolumns=2,my.ylab=my.ylab,
+    which.ps=which.ps,cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T),my.cex.lab=1,set.cols=F)
>  NHPP.fitrange.plot(model.3,ngraphs=2,par.order=c(8,3),ncolumns=2,my.ylab=my.ylab,
+    which.ps=which.ps,cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T),my.cex.lab=1,set.cols=F)
```



4.2 NHP covariate selection and checking

We illustrate covariate selection for model 1 using $\tau = 0.9$. For the purposes of comparing models the choice of block size, i.e. the number of observations per block,n_B say, is arbitrary. Following Wadsworth et al. (2010) we set n_B such that the total number of blocks, n/n_B, is equal to the (approximate) number of threshold excesses $n(1-\tau)$, i.e. that $n_B = 1/(1-\tau)$ This can speed up fitting by making the parameter estimators less strongly associated.

```
> p.exc <- 0.10 # set the probability of threshold exceedance (i.e. 90% conditional quantile)
> nB <- 1/p.exc
```

```
> res <- NULL
> which.comp <- NULL
> # none vs. NAO
> M1 <- NHPP.fit(Hs,xdat=xdat,p.exc=p.exc,nB=nB)
```

```
maximized log-likelihood =  -2419.5292
          MLE      SE MLE.SE.RATIO
mu     7.0975 0.07733        91.78
sigma  1.7464 0.09754        17.91
xi    -0.1172 0.03399        -3.45
```

```
>   temp <- NHPP.add(M1,mul=1)$LRT
```

```
Smaller model
-------------
maximized log-likelihood =  -2419.529
        MLE    SE MLE.SE.RATIO
mu     7.10 0.077         91.8
sigma  1.75 0.098         17.9
xi    -0.12 0.034         -3.4

Larger model
------------
maximized log-likelihood =  -2360.221
         MLE    SE MLE.SE.RATIO
mu.0    6.85 0.088         77.7
mu.NAO  1.12 0.117          9.5
sigma   1.78 0.092         19.4
xi     -0.16 0.031         -5.0

  LR.test.stat df p.value
1        118.6  1       0
```

```
> init.ests <- add.zeros(M1$mle,2)
> M1.NAO <- NHPP.fit(Hs,xdat=xdat,mul=1,p.exc=p.exc,nB=nB,init.ests=init.ests)
```

```
maximized log-likelihood =  -2384.8683
           MLE      SE MLE.SE.RATIO
mu.0    6.9879 0.07197       97.091
mu.NAO  0.9166 0.08983       10.203
sigma   1.6214 0.09387       17.272
xi     -0.1182 0.03731       -3.167
```

```
>   temp <- cbind(temp,NHPP.drop(M1.NAO,mul=1)$LRT)
```

```
Larger model
-------------
```

```
maximized log-likelihood =  -2384.868
          MLE    SE MLE.SE.RATIO
mu.0    6.99 0.072         97.1
mu.NAO  0.92 0.090         10.2
sigma   1.62 0.094         17.3
xi     -0.12 0.037         -3.2

Smaller model
------------
maximized log-likelihood =  -2430.398
         MLE    SE MLE.SE.RATIO
mu     6.93 0.086         80.2
sigma  1.94 0.093         21.0
xi    -0.15 0.029         -5.3

  LR.test.stat df p.value
1        91.06  1       0
  LR.test.stat df p.value
1        91.06  1       0


> res <- rbind(res,temp)
> which.comp <- c(which.comp,"none vs. NAO")
> # none vs. S
>   temp <- NHPP.add(M1,mul=2:3)$LRT


Smaller model
-------------
maximized log-likelihood =  -2419.529
         MLE    SE MLE.SE.RATIO
mu     7.10 0.077         91.8
sigma  1.75 0.098         17.9
xi    -0.12 0.034         -3.4

Larger model
------------
maximized log-likelihood =  -2187.847
            MLE    SE MLE.SE.RATIO
mu.0      6.16 0.119         52.0
mu.cos.1  2.81 0.180         15.6
mu.sin.1  0.17 0.118          1.4
sigma     1.73 0.075         22.9
xi       -0.19 0.027         -6.9

  LR.test.stat df p.value
1        463.4  2       0


> M1.season <- NHPP.fit(Hs,xdat=xdat,mul=2:3,p.exc=p.exc,nB=nB)


maximized log-likelihood =  -2291.7838
               MLE      SE MLE.SE.RATIO
mu.0      6.36095 0.05396      117.886
mu.cos.1  2.66564 0.07714       34.554
mu.sin.1  0.19066 0.07616        2.503
sigma     1.21912 0.07973       15.290
xi       -0.01422 0.04838       -0.294


> init.ests <- add.zeros(M1$mle,2:3)
> M1.season <- NHPP.fit(Hs,xdat=xdat,mul=2:3,p.exc=p.exc,nB=nB,init.ests=init.ests)
```

```
maximized log-likelihood =  -2291.782
             MLE      SE MLE.SE.RATIO
mu.0      6.3591 0.05403     117.7044
mu.cos.1  2.6682 0.07727      34.5300
mu.sin.1  0.1915 0.07624       2.5111
sigma     1.2197 0.07971      15.3012
xi       -0.0156 0.04812      -0.3243
```

>   temp <- cbind(temp,NHPP.drop(M1.season,mu1=2:3)$LRT)

```
Larger model
-------------
maximized log-likelihood =  -2291.782
            MLE    SE MLE.SE.RATIO
mu.0      6.359 0.054       117.70
mu.cos.1  2.668 0.077        34.53
mu.sin.1  0.191 0.076         2.51
sigma     1.220 0.080        15.30
xi       -0.016 0.048        -0.32

Smaller model
------------
maximized log-likelihood =  -2515.18
        MLE    SE MLE.SE.RATIO
mu     5.97 0.127           47
sigma  2.79 0.113           25
xi    -0.25 0.024          -10

  LR.test.stat df p.value
1        446.8  2       0
  LR.test.stat df p.value
1        446.8  2       0
```

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"none vs. S")
> # NAO vs. NAO+S
>   temp <- NHPP.add(M1.NAO,mu1=2:3)$LRT

```
Smaller model
-------------
maximized log-likelihood =  -2384.868
         MLE    SE MLE.SE.RATIO
mu.0    6.99 0.072         97.1
mu.NAO  0.92 0.090         10.2
sigma   1.62 0.094         17.3
xi     -0.12 0.037         -3.2

Larger model
------------
maximized log-likelihood =  -2169.391
            MLE    SE MLE.SE.RATIO
mu.0      6.195 0.107        58.07
mu.NAO    0.787 0.091         8.62
mu.cos.1  2.500 0.165        15.15
mu.sin.1  0.067 0.109         0.61
sigma     1.611 0.072        22.53
xi       -0.189 0.028        -6.81

  LR.test.stat df p.value
1          431  2       0
```

```
> # estimates from threshold stability plot
> init.ests <- add.zeros(M1.season$mle,2)
> M1.NAO.season <- NHPP.fit(Hs,xdat=xdat,mul=1:3,p.exc=p.exc,nB=nB,init.ests=init.ests)

maximized log-likelihood =  -2264.0611
              MLE      SE MLE.SE.RATIO
mu.0      6.33420 0.05226     121.2077
mu.NAO    0.43423 0.06673       6.5068
mu.cos.1  2.59795 0.07437      34.9347
mu.sin.1  0.18576 0.07371       2.5203
sigma     1.17428 0.07779      15.0953
xi       -0.01874 0.04905      -0.3821

>   temp <- cbind(temp,NHPP.drop(M1.NAO.season,mul=2:3)$LRT)

Larger model
-------------
maximized log-likelihood =  -2264.061
            MLE    SE MLE.SE.RATIO
mu.0      6.334 0.052       121.21
mu.NAO    0.434 0.067         6.51
mu.cos.1  2.598 0.074        34.93
mu.sin.1  0.186 0.074         2.52
sigma     1.174 0.078        15.10
xi       -0.019 0.049        -0.38

Smaller model
------------
maximized log-likelihood =  -2483.327
         MLE    SE MLE.SE.RATIO
mu.0    5.93 0.124         47.7
mu.NAO  0.90 0.145          6.2
sigma   2.73 0.109         25.0
xi     -0.28 0.021        -13.1

  LR.test.stat df p.value
1        438.5  2       0
  LR.test.stat df p.value
1        438.5  2       0

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"NAO vs. NAO+S")
> # S vs. NAO+S
>   temp <- NHPP.add(M1.season,mul=1)$LRT

Smaller model
-------------
maximized log-likelihood =  -2291.782
            MLE    SE MLE.SE.RATIO
mu.0      6.359 0.054       117.70
mu.cos.1  2.668 0.077        34.53
mu.sin.1  0.191 0.076         2.51
sigma     1.220 0.080        15.30
xi       -0.016 0.048        -0.32

Larger model
------------
maximized log-likelihood =  -2270.143
           MLE    SE MLE.SE.RATIO
```

```
mu.0      6.28 0.061        103.5
mu.cos.1  2.64 0.081         32.5
mu.sin.1  0.16 0.081          2.0
mu.NAO    0.49 0.083          5.9
sigma     1.29 0.080         16.1
xi       -0.07 0.044         -1.6

  LR.test.stat df   p.value
1        43.28  1 4.747e-11

>   temp <- cbind(temp,NHPP.drop(M1.NAO.season,mul=1)$LRT)

Larger model
-------------
maximized log-likelihood =  -2264.061
            MLE    SE MLE.SE.RATIO
mu.0      6.334 0.052       121.21
mu.NAO    0.434 0.067         6.51
mu.cos.1  2.598 0.074        34.93
mu.sin.1  0.186 0.074         2.52
sigma     1.174 0.078        15.10
xi       -0.019 0.049        -0.38

Smaller model
------------
maximized log-likelihood =  -2284.472
            MLE    SE MLE.SE.RATIO
mu.0      6.314 0.058        109.2
mu.cos.1  2.642 0.083         31.8
mu.sin.1  0.215 0.081          2.6
sigma     1.300 0.075         17.4
xi       -0.047 0.043         -1.1

  LR.test.stat df   p.value
1        40.82  1 1.667e-10
  LR.test.stat df   p.value
1        40.82  1 1.667e-10

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"S vs. NAO+S")
> # NAO+S vs NAO+S+NAO:S
>   temp <- NHPP.add(M1.NAO.season,mul=4:5)$LRT

Smaller model
-------------
maximized log-likelihood =  -2264.061
            MLE    SE MLE.SE.RATIO
mu.0      6.334 0.052       121.21
mu.NAO    0.434 0.067         6.51
mu.cos.1  2.598 0.074        34.93
mu.sin.1  0.186 0.074         2.52
sigma     1.174 0.078        15.10
xi       -0.019 0.049        -0.38

Larger model
------------
maximized log-likelihood =  -2249.092
                MLE    SE MLE.SE.RATIO
mu.0          6.282 0.055       114.06
mu.NAO        0.438 0.068         6.41
```

```
mu.cos.1      2.568 0.077        33.28
mu.sin.1      0.164 0.075         2.18
mu.NAO.cos.1  0.519 0.100         5.17
mu.NAO.sin.1  0.025 0.093         0.27
sigma         1.182 0.077        15.40
xi           -0.024 0.049        -0.49

  LR.test.stat df   p.value
1        29.94  2 3.156e-07

> # estimates from threshold stability plot
> init.ests <- add.zeros(M1.NAO.season$mle,5:6)
> M1.NAO.season.int <- NHPP.fit(Hs,xdat=xdat,mul=1:5,p.exc=p.exc,nB=nB,init.ests=init.ests)

maximized log-likelihood =  -2266.1157
                  MLE      SE MLE.SE.RATIO
mu.0          6.25761 0.05483     114.1297
mu.NAO        0.55666 0.06886       8.0839
mu.cos.1      2.56234 0.07795      32.8716
mu.sin.1      0.14912 0.07716       1.9326
mu.NAO.cos.1  0.54276 0.09817       5.5285
mu.NAO.sin.1  0.02525 0.09592       0.2632
sigma         1.22554 0.07912      15.4898
xi           -0.05049 0.04684      -1.0779

>   temp <- cbind(temp,NHPP.drop(M1.NAO.season.int,mul=4:5)$LRT)

Larger model
-------------
maximized log-likelihood =  -2266.116
                MLE    SE MLE.SE.RATIO
mu.0          6.258 0.055       114.13
mu.NAO        0.557 0.069         8.08
mu.cos.1      2.562 0.078        32.87
mu.sin.1      0.149 0.077         1.93
mu.NAO.cos.1  0.543 0.098         5.53
mu.NAO.sin.1  0.025 0.096         0.26
sigma         1.226 0.079        15.49
xi           -0.050 0.047        -1.08

Smaller model
------------
maximized log-likelihood =  -2280.146
            MLE    SE MLE.SE.RATIO
mu.0      6.245 0.060        103.5
mu.NAO    0.586 0.075          7.8
mu.cos.1  2.610 0.086         30.4
mu.sin.1  0.166 0.085          2.0
sigma     1.352 0.077         17.6
xi       -0.098 0.039         -2.5

  LR.test.stat df   p.value
1        28.06  2 8.071e-07
  LR.test.stat df   p.value
1        28.06  2 8.071e-07

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"NAO+S vs NAO+S+NAO:S")
> # NAO+S+NAO:S vs NAO+S+NAO:S + sigma:NAO
> temp <- NHPP.add(M1.NAO.season.int,sigl=1)$LRT # add seasonal sigma
```

```
Smaller model
-------------
maximized log-likelihood =  -2266.116
                MLE    SE MLE.SE.RATIO
mu.0          6.258 0.055       114.13
mu.NAO        0.557 0.069         8.08
mu.cos.1      2.562 0.078        32.87
mu.sin.1      0.149 0.077         1.93
mu.NAO.cos.1  0.543 0.098         5.53
mu.NAO.sin.1  0.025 0.096         0.26
sigma         1.226 0.079        15.49
xi           -0.050 0.047        -1.08

Larger model
------------
maximized log-likelihood =  -2264.909
                MLE    SE MLE.SE.RATIO
mu.0          6.256 0.055        114.0
mu.NAO        0.545 0.068          8.0
mu.cos.1      2.566 0.078         32.9
mu.sin.1      0.147 0.077          1.9
mu.NAO.cos.1  0.553 0.097          5.7
mu.NAO.sin.1  0.019 0.095          0.2
sigma.0       1.228 0.079         15.6
sigma.NAO     0.083 0.053          1.6
xi           -0.056 0.046         -1.2

  LR.test.stat df p.value
1        2.413  1  0.1204

> init.ests <- add.zeros(M1.NAO.season.int$mle,8)
> M1.NAO.season.int.sigma.NAO <- NHPP.fit(Hs,xdat=xdat,mul=1:5,sigl=1,p.exc=p.exc,nB=nB,
                                                                  init.ests=init.ests)
maximized log-likelihood =  -2264.9094
                  MLE      SE MLE.SE.RATIO
mu.0          6.25624 0.05486     114.0411
mu.NAO        0.54506 0.06829       7.9821
mu.cos.1      2.56594 0.07801      32.8941
mu.sin.1      0.14724 0.07713       1.9089
mu.NAO.cos.1  0.55300 0.09678       5.7138
mu.NAO.sin.1  0.01883 0.09456       0.1991
sigma.0       1.22841 0.07881      15.5870
sigma.NAO     0.08343 0.05264       1.5849
xi           -0.05641 0.04597      -1.2271

> temp <- cbind(temp,NHPP.drop(M1.NAO.season.int.sigma.NAO,sigl=1)$LRT)

Larger model
-------------
maximized log-likelihood =  -2264.909
                MLE    SE MLE.SE.RATIO
mu.0          6.256 0.055        114.0
mu.NAO        0.545 0.068          8.0
mu.cos.1      2.566 0.078         32.9
mu.sin.1      0.147 0.077          1.9
mu.NAO.cos.1  0.553 0.097          5.7
mu.NAO.sin.1  0.019 0.095          0.2
sigma.0       1.228 0.079         15.6
sigma.NAO     0.083 0.053          1.6
xi           -0.056 0.046         -1.2
```

```
Smaller model
------------
maximized log-likelihood =  -2266.112
                 MLE    SE MLE.SE.RATIO
mu.0           6.257 0.055       113.73
mu.NAO         0.559 0.069         8.09
mu.cos.1       2.562 0.078        32.76
mu.sin.1       0.147 0.077         1.89
mu.NAO.cos.1   0.545 0.098         5.54
mu.NAO.sin.1   0.021 0.096         0.22
sigma          1.229 0.079        15.51
xi            -0.053 0.046        -1.15

  LR.test.stat df p.value
1        2.405  1  0.1209
  LR.test.stat df p.value
1        2.405  1  0.1209

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"NAO+S+NAO:S vs NAO+S+NAO:S + sigma:NAO")
> # NAO+S+NAO:S vs NAO+S+NAO:S + sigma:S
> temp <- NHPP.add(M1.NAO.season.int,sigl=2:3)$LRT # add seasonal sigma

Smaller model
-------------
maximized log-likelihood =  -2266.116
                 MLE    SE MLE.SE.RATIO
mu.0           6.258 0.055       114.13
mu.NAO         0.557 0.069         8.08
mu.cos.1       2.562 0.078        32.87
mu.sin.1       0.149 0.077         1.93
mu.NAO.cos.1   0.543 0.098         5.53
mu.NAO.sin.1   0.025 0.096         0.26
sigma          1.226 0.079        15.49
xi            -0.050 0.047        -1.08

Larger model
------------
maximized log-likelihood =  -2263.575
                 MLE    SE MLE.SE.RATIO
mu.0           6.257 0.055       113.65
mu.NAO         0.557 0.069         8.06
mu.cos.1       2.574 0.078        33.11
mu.sin.1       0.165 0.078         2.13
mu.NAO.cos.1   0.529 0.099         5.36
mu.NAO.sin.1   0.015 0.096         0.15
sigma.0        1.229 0.080        15.45
sigma.cos.1    0.163 0.081         2.00
sigma.sin.1   -0.045 0.068        -0.66
xi            -0.053 0.047        -1.11

  LR.test.stat df p.value
1        5.081  2 0.07881

> init.ests <- add.zeros(M1.NAO.season.int$mle,8:9)
> M1.NAO.season.int.sigma.S <- NHPP.fit(Hs,xdat=xdat,mul=1:5,sigl=2:3,p.exc=p.exc,nB=nB,
                                                            init.ests=init.ests)

maximized log-likelihood =  -2263.5749
                   MLE      SE MLE.SE.RATIO
```

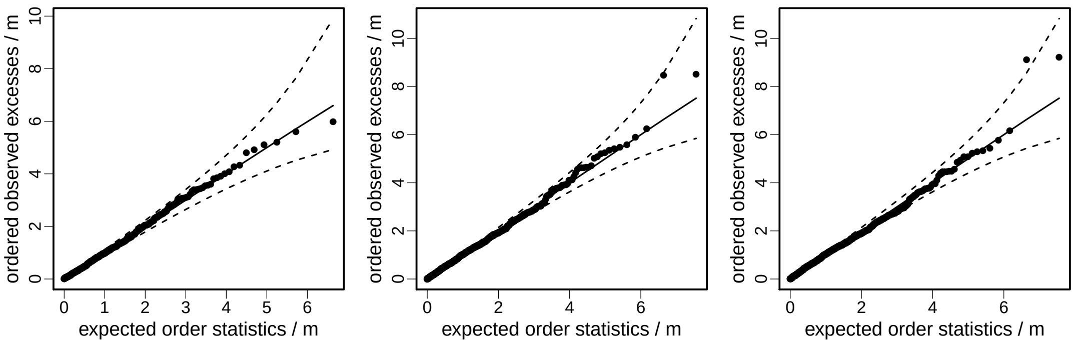
```
mu.0          6.25652 0.05505     113.6489
mu.NAO        0.55707 0.06914       8.0570
mu.cos.1      2.57376 0.07773      33.1117
mu.sin.1      0.16541 0.07774       2.1277
mu.NAO.cos.1  0.52880 0.09868       5.3589
mu.NAO.sin.1  0.01458 0.09567       0.1524
sigma.0       1.22923 0.07955      15.4521
sigma.cos.1   0.16282 0.08139       2.0005
sigma.sin.1  -0.04486 0.06830      -0.6568
xi           -0.05279 0.04737      -1.1143

> temp <- cbind(temp,NHPP.drop(M1.NAO.season.int.sigma.S,sig=2:3)$LRT)

Larger model
-------------
maximized log-likelihood =  -2263.575
                MLE    SE MLE.SE.RATIO
mu.0          6.257 0.055       113.65
mu.NAO        0.557 0.069         8.06
mu.cos.1      2.574 0.078        33.11
mu.sin.1      0.165 0.078         2.13
mu.NAO.cos.1  0.529 0.099         5.36
mu.NAO.sin.1  0.015 0.096         0.15
sigma.0       1.229 0.080        15.45
sigma.cos.1   0.163 0.081         2.00
sigma.sin.1  -0.045 0.068        -0.66
xi           -0.053 0.047        -1.11

Smaller model
------------
maximized log-likelihood =  -2266.112
                MLE    SE MLE.SE.RATIO
mu.0          6.257 0.055       113.75
mu.NAO        0.559 0.069         8.09
mu.cos.1      2.563 0.078        32.77
mu.sin.1      0.147 0.077         1.89
mu.NAO.cos.1  0.546 0.098         5.54
mu.NAO.sin.1  0.022 0.096         0.23
sigma         1.229 0.079        15.51
xi           -0.053 0.047        -1.14

  LR.test.stat df p.value
1        5.074  2  0.0791
  LR.test.stat df p.value
1        5.074  2  0.0791

> res <- rbind(res,temp)
> which.comp <- c(which.comp,"NAO+S+NAO:S vs NAO+S+NAO:S + sigma:S")
> # Table 1
> res.LATEX <- round(res[,c(2,4,6,1,3)],3) # reorder the columns to produce table 1

> p.exc <- 0.1
> init.ests <- model.1$mles[abs(model.1$p.exc-p.exc)<1e-10,]
> M1.NS.int <- NHPP.fit(Hs,xdat=xdat,mul=1:5,p.exc=p.exc,nB=1,init.ests=init.ests,
+    cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T))

maximized log-likelihood =  -2266.1122
           MLE     SE MLE.SE.RATIO
mu.0     3.239  0.344         9.41
```

```
mu.NAO   0.559  0.069         8.09
mu.A1    2.567  0.078        32.75
mu.phi1  3.321  1.752         1.90
mu.A2    0.546  0.099         5.52
mu.phi2  2.353 10.217         0.23
sigma    1.394  0.225         6.19
xi      -0.054  0.046        -1.16

> p.exc <- 0.25
> init.ests <- model.2$mles[abs(model.2$p.exc-p.exc)<1e-10,]
> M2.NS.int <- NHPP.fit(Hs,xdat=xdat,mul=1:5,p.exc=p.exc,nB=1,init.ests=init.ests,mulink=exp,
+    sig.propto.mu=T,cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T))

maximized log-likelihood =  -4688.9028
           MLE     SE MLE.SE.RATIO
mu.0     0.975  0.042        23.04
mu.NAO   0.083  0.018         4.55
mu.A1    0.425  0.022        19.12
mu.phi1  3.655  2.776         1.32
mu.A2    0.066  0.028         2.39
mu.phi2  9.789 21.251         0.46
sigma    0.584  0.038        15.36
xi      -0.068  0.024        -2.84

> p.exc <- 0.25
> init.ests <- model.3$mles[abs(model.3$p.exc-p.exc)<1e-10,]
> M3.NS.int <- NHPP.fit(log(Hs),xdat=xdat,mul=1:5,p.exc=p.exc,nB=1,init.ests=init.ests,
+    cs.to.ap=matrix(c(3,4,5,6),ncol=2,byrow=T))

maximized log-likelihood =  -2402.922
           MLE      SE MLE.SE.RATIO
mu.0     1.091  0.0232        46.93
mu.NAO   0.084  0.0093         9.05
mu.A1    0.429  0.0114        37.74
mu.phi1  3.785  1.4080         2.69
mu.A2    0.069  0.0140         4.89
mu.phi2 10.367 10.6764         0.97
sigma    0.384  0.0219        17.58
xi      -0.228  0.0189       -12.03

> # Table 2
>
> paste(signif(M1.NS.int$mle.amp.phase,3),sep=" ",collapse="&")

[1] "3.24&0.559&2.57&3.32&0.546&2.35&1.39&-0.0541"

> paste(signif(M1.NS.int$se.amp.phase,4),sep=" ",collapse="&")

[1] "0.3443&0.06917&0.07838&1.752&0.09885&10.22&0.2253&0.04648"

> paste(signif(M2.NS.int$mle.amp.phase,3),sep=" ",collapse="&")

[1] "0.975&0.0827&0.425&3.65&0.0661&9.79&0.584&-0.0684"

> paste(signif(M2.NS.int$se.amp.phase,3),sep=" ",collapse="&")
```

```
[1] "0.0423&0.0182&0.0222&2.78&0.0276&21.3&0.0381&0.0241"

> paste(signif(M3.NS.int$mle.amp.phase,3),sep=" ",collapse="&")

[1] "1.09&0.0843&0.429&3.78&0.0686&10.4&0.384&-0.228"

> paste(signif(M3.NS.int$se.amp.phase,3),sep=" ",collapse="&")

[1] "0.0232&0.00931&0.0114&1.41&0.014&10.7&0.0219&0.0189"
```

4.3 Residual QQ plots

```
> par(mfrow=c(1,3),mar=c(3,3.25,1.5,0.75),lwd=3,cex.lab=1.25,cex.axis=1.1,cex=1.5,mgp=c(3,0.5,0))
> NHPP.residual.plot(M1.NS.int,ann=F,lwd=2.5)

[1] "calculating 2.5% envelope"
[1] "calculating 50% envelope"
[1] "calculating 97.5% envelope"
14 of 508 ( 2.8 % ) residuals outside 95% envelope

> NHPP.residual.plot(M2.NS.int,ann=F,lwd=2.5)

[1] "calculating 2.5% envelope"
[1] "calculating 50% envelope"
[1] "calculating 97.5% envelope"
0 of 1273 ( 0 % ) residuals outside 95% envelope

> NHPP.residual.plot(M3.NS.int,ann=F,lwd=2.5)

[1] "calculating 2.5% envelope"
[1] "calculating 50% envelope"
[1] "calculating 97.5% envelope"
39 of 1273 ( 3.1 % ) residuals outside 95% envelope
```



References

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Wadsworth, J. L., J. A. Tawn, and P. Jonathan (2010). Accounting for choice of measurement scale in extreme value modelling. *4*, 1558–1578.