

Analysis of the error in using insurance mortality to price long term care products

Guy Rolland Rasoanaivo, Jeyaraj Vadiveloo, Charles Vinsonhaler and
Nalini Ravishanker

Abstract: Long term care (LTC) pricing requires the use of underlying experience data of mortality and lapse rates of healthy insureds, LTC incidence rates, LTC utilization rates, and LTC termination rates from recovery or death. It is important that the pricing process uses the most up-to-date industry tables, modified for a company's own experience and anticipated changes due to improvements in medical science and technology. While companies have recognized this fact, less attention has been paid on the mortality of healthy insureds. It is customary for companies to use either industry experience tables or their own experience of mortality on life insurance in LTC pricing. Therein lies the subtle error that results: Life insurance mortality does not distinguish between death occurring from a prior healthy state, or death occurring from a prior disabled state. In LTC pricing, we need to isolate the mortality rate of healthy insureds i.e., only consider deaths occurring from a prior healthy state. In this paper, we analyze the impact of this error for the two forms of LTC coverage - the stand-alone LTC product and LTC as a rider to a life insurance product. The paper demonstrates the following: (a) how to create the theoretically correct mortality table for LTC pricing from a life insurance mortality table and mortality assumption for disabled lives; (b) a mathematical proof on the pricing impact of this error; and (c) some examples on the magnitude of this error for selected ages and different mortality assumptions for disabled lives.

Key Words: corrected mortality table; disability; LTC as rider to a life insurance; stand-alone LTC.

1 Introduction

Long term care (LTC) pricing requires the use of underlying experience data of mortality and lapse rates of healthy insureds, LTC incidence rates, LTC utilization rates, and LTC termination rates from recovery or death. Clearly this is a dynamic process, particularly for the LTC incidence, utilization and termination rates, because of advances in medical science and technology. It is therefore important that a pricing process use the most up-to-date industry tables, modified for a company's own experience and anticipated changes due to improvements in medical science.

While companies have recognized this fact, less attention has been paid to the mortality of healthy insureds. It is customary for companies to use either industry experience tables or their own experience of mortality on life insurance in LTC pricing. Therein lies the subtle error that results: life insurance mortality does not distinguish between death occurring from a prior healthy state, or death occurring from a prior disabled state. In this paper, we analyze the impact of this error for the two forms of LTC coverage - the stand-alone LTC product and LTC as a rider to a life insurance product. The paper demonstrates the following: (a) how to create the theoretically correct mortality table for LTC pricing from a life insurance mortality table and mortality assumption for disabled lives; (b) a mathematical derivation of the pricing impact of this error; and (c) some examples on the magnitude of the error for selected ages and different mortality assumptions for disabled lives.

2 Current Deterministic LTC Rider Policies

The concept of active life expectancy and the end of active life has been defined as a loss of independence in the Activities of Daily Living (ADL). The ADLs considered are (i) bathing, (ii) dressing, (iii) toileting, (iv) transferring, (v) continence, and (vi) feeding. Benefit eligibility criteria used are deficiency in performing Activities of Daily Living (ADLs) and determination of cognitive impairment (CI). Cognitive impairment (CI) is a constituent feature of severe depression and schizophrenia or any health condition which affects the ability to think, concentrate, formulate ideas, reason, and remember.

An **LTC Rider** in a life insurance policy is defined as follows. An individual buys the policy at an initial age, at which time the individual is assumed

to be healthy. While the policy is in force and the policyholder is disabled (more than 2 ADLs in most cases), with duration of disability greater than the waiting period, a level LTC Benefit is payable monthly. The policy ceases when the individual dies or reaches the age limit of the benefit payment, or when the total benefit paid reaches the maximum allowed, whichever occurs sooner. Upon death, the difference between the face amount of the policy and the total LTC benefit paid to date is payable as a death benefit. An LTC Stand-Alone has the same features as the LTC Rider except that there is no payment at death (Bowers *et al*, 1986).

The **LTC Rider Cost** is expressed as the difference of a random variable for the present value of LTC benefits paid and any remaining death benefit, less a random variable for the present value of only the unreduced death benefit paid at death. Note that total benefits paid are the same with or without the Rider. The only difference is that with the rider, benefits are paid earlier. For the LTC Stand-Alone, the cost random variable is the present value of LTC benefits paid.

In pricing LTC policies, standard industry practice uses a deterministic model such as the one we now describe. We assume that there is no recovery from disability, i.e., if an individual starts to receive LTC benefits, the only two possible transition states are staying disabled or death. Consider a person age x who just been issued a life insurance policy with LTC benefits as a rider. Let ω be the terminal age at death and v be the discount factor, $0 < v < 1$. Let $q_x^{(T)}$ denote the probability of dying for *all lives* at age x . A typical industry approach to determine the cost of the rider uses the formula:

$$\begin{aligned}
C_{(ri)} &= \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} q_{x+t}^{(T)} v^{t+1} F A + \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} v^{t+1+u} B_{x+t+u+1} \\
&+ \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} D B_{x+t+u+1} \\
&- \left[\sum_{t=0}^{\omega-x} {}_t p_x^{(T)} q_{x+t}^{(T)} v^{t+1} F A + \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} F A \right]
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
{}_t p_x^{(T)} &= {}_{t-1} p_x^{(T)} (1 - q_{x+t-1}^{(T)} - r_{x+t-1} - q_{x+t-1}^{(w)}) \\
{}_t p_x^{(r)} &= 1 - {}_t q_x^{(r)}
\end{aligned}$$

$q_x^{(T)}$: Probability of dying for all lives at age x ,

r_x : Probability a healthy life becomes disabled at age x ,

$q_x^{(r)}$: Probability a disabled life dies at age x ,

$q_x^{(w)}$: Probability a healthy life withdraws at age x ,

FA : Face Amount

B_{x+t} : LTC Benefit paid at age $x + t$ after disability occurs

DB_{x+t} : Residual death benefit paid at age $x + t$ for a disabled life who dies.

The first term on the right side of (1) covers the expected cost of the individual who dies without getting disabled. The second term covers the expected cost of just the disability portion of the benefit for the individual who gets disabled. The third term covers the expected cost of the death benefit to the individual who gets disabled. The fourth and last terms represent the expected cost of life insurance without the rider where the individual gets the full face amount whether he dies from a healthy state or from a state of impairment. A simplification of (1) yields:

$$\begin{aligned}
C_{(ri)} &= \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} Ben_{x+t} \\
&\quad - \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} FA
\end{aligned} \tag{2}$$

where

$$Ben_{x+t} = \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} v^{t+1+u} \left(B_{x+t+u+1} + q_{x+t+u}^{(r)} DB_{x+t+u+1} \right).$$

Similarly, the full expression for the cost of the LTC stand-alone is

$$C_{(si)} = \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} v^{t+1+u} B_{x+t+u+1}. \tag{3}$$

The right side of (3) is similar to the second term on the right side of equation (1), since the benefit payment only occurs when the individual is disabled and no payment is made upon death (TSA Report, 1995).

3 Deterministic Pricing Formulas

We saw in Section 2 that for pricing policies, the industry has typically used $q_x^{(T)}$, probability of dying for all lives at age x , instead of $q_x^{(h)}$, the mortality for healthy lives. The industry formula (2) is incorrect. We present the correct formula, and compare the current industry formula with the theoretically correct formula that we propose. We also quantify the difference numerically.

3.1 Theoretically Correct Deterministic Formula

We argue for the use of $q_x^{(h)}$, the mortality rate for *healthy lives*, in place of $q_x^{(T)}$ in (1):

$$\begin{aligned}
C_{(rc)} &= \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} q_{x+t}^{(h)} v^{t+1} F A + \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} v^{t+1+u} B_{x+t+u+1} \\
&+ \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} D B_{x+t+u+1} \\
&- \left[\sum_{t=0}^{\omega-x} {}_t p_x^{(h)} q_{x+t}^{(h)} v^{t+1} F A + \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} F A \right]
\end{aligned} \tag{4}$$

where

$${}_t p_x^{(h)} = {}_{t-1} p_x^{(h)} (1 - q_{x+t-1}^{(h)} - r_{x+t-1} - q_{x+t-1}^{(w)})$$

After simplification, (4) becomes (compare (2)):

$$\begin{aligned}
C_{(rc)} &= \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} B e n_{x+t} \\
&- \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} F A
\end{aligned} \tag{5}$$

where $B e n_{x+t}$ was defined earlier.

For the LTC Stand-Alone, the rider formula is again similar in form to the current industry formula, except that ${}_t p_x^{(h)}$ is used in place of ${}_t p_x^{(T)}$:

$$C_{(sc)} = \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} v^{t+1+u} B_{x+t+u+1}. \tag{6}$$

3.2 Comparison of Current and Revised Industry Pricing Models

In this section, we quantify the extent to which the current industry pricing procedure underprices. We make a strong argument for our revised rider, using $q_x^{(h)}$ in place of $q_x^{(T)}$. We first describe a method for generating the probability of death for healthy lives. Let

- $l_x^{(T)}$ be the number of all lives at age x ,
- $l_x^{(h)}$ be the number of healthy lives at age x ,
- $l_x^{(r)}$ be the number of disabled lives at age x ,
- $d_x^{(T)}$ be the number of deaths for all lives at age x ,
- $d_x^{(r)}$ be the number of deaths for disabled lives at age x ,
- $d_x^{(h)}$ be the number of deaths for healthy lives at age x ,
- $q_x^{(T)}$ be the probability of dying for all lives at age x ,
- r_x be the probability of becoming disabled for healthy lives at age x ,
- $q_x^{(r)}$ be the probability of dying for disabled lives at age x ,
- $q_x^{(w)}$ be the probability of withdrawal for healthy lives at age x ,

We assume withdrawals and impairments occur at the beginning of the period, in that order, and that deaths occur at the end of the period. We have the following relationships

$$d_x^{(T)} = [l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)}] q_x^{(T)} \quad (7)$$

$$d_x^{(r)} = l_x^{(h)} (1 - q_x^{(w)}) r_x q_x^{(r)} + l_x^{(r)} q_x^{(r)} \quad (8)$$

$$d_x^{(h)} = l_x^{(h)} (1 - r_x) (1 - q_x^{(w)}) q_x^{(h)} \quad (9)$$

$$d_x^{(T)} = d_x^{(r)} + d_x^{(h)}. \quad (10)$$

Substituting the expressions in (7), (8) and (9) into (10), we have

$$\begin{aligned} [l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)}] q_x^{(T)} &= l_x^{(h)} (1 - q_x^{(w)}) r_x q_x^{(r)} + l_x^{(r)} q_x^{(r)} \\ &+ l_x^{(h)} (1 - r_x) (1 - q_x^{(w)}) q_x^{(h)}. \end{aligned} \quad (11)$$

Solving (11) for $q_x^{(h)}$, the probability of death for healthy lives at age x is:

$$q_x^{(h)} = \frac{l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)}}{l_x^{(h)} (1 - r_x) (1 - q_x^{(w)})} q_x^{(T)} - \frac{l_x^{(h)} (1 - q_x^{(w)}) r_x + l_x^{(r)}}{l_x^{(h)} (1 - r_x) (1 - q_x^{(w)})} q_x^{(r)} \quad (12)$$

with

$$l_x^{(r)} = l_{x-1}^{(r)} (1 - q_{x-1}^{(r)}) + l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) r_{x-1} (1 - q_{x-1}^{(r)}) \quad (13)$$

$$l_x^{(h)} = l_{x-1}^{(h)} (1 - r_{x-1}) (1 - q_{x-1}^{(w)}) (1 - q_{x-1}^{(h)}) \quad (14)$$

$$l_x^{(T)} = \left[l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) + l_{x-1}^{(r)} \right] (1 - q_{x-1}^{(T)}) \quad (15)$$

We begin discussion of the pricing methods with an intuitively obvious fact.

Remark 1 *As defined by (13)-(15), $l_x^{(T)} = l_x^{(r)} + l_x^{(h)}$. This shows consistency in the life table relationships.*

Indeed, from (13) and (14) we have:

$$\begin{aligned} l_x^{(r)} + l_x^{(h)} &= l_{x-1}^{(r)} + l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) r_{x-1} + l_{x-1}^{(h)} (1 - r_{x-1}) (1 - q_{x-1}^{(w)}) \\ &\quad - \underbrace{\left[q_{x-1}^{(r)} (l_{x-1}^{(r)} + l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) r_{x-1}) + q_{x-1}^{(h)} l_{x-1}^{(h)} (1 - r_{x-1}) (1 - q_{x-1}^{(w)}) \right]}_{\left[l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) + l_{x-1}^{(r)} \right] q_{x-1}^{(T)} \quad \text{[from (11)]}} \end{aligned}$$

Thus,

$$\begin{aligned} l_x^{(r)} + l_x^{(h)} &= l_{x-1}^{(r)} + l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) - \left[l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) + l_{x-1}^{(r)} \right] q_{x-1}^{(T)} \\ &= l_{x-1}^{(h)} (1 - q_{x-1}^{(w)}) (1 - q_{x-1}^{(T)}) + l_{x-1}^{(r)} (1 - q_{x-1}^{(T)}) \\ &= l_x^{(T)} \quad \text{[from (15)]} \end{aligned}$$

Proposition 1 (a) *With the above definitions, $q_x^{(h)} \leq q_x^{(T)}$.*

(b) *For all $t \geq 1$, ${}_t p_x^{(h)} \geq {}_t p_x^{(T)}$.*

Proof. (a) From (12), we have

$$q_x^{(T)} = \frac{l_x^{(h)} (1 - r_x) (1 - q_x^{(w)})}{l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)}} q_x^{(h)} + \frac{l_x^{(h)} (1 - q_x^{(w)}) r_x + l_x^{(r)}}{l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)}} q_x^{(r)}.$$

We can see that

$$l_x^{(h)} (1 - r_x) (1 - q_x^{(w)}) + l_x^{(h)} (1 - q_x^{(w)}) r_x + l_x^{(r)} = l_x^{(h)} (1 - q_x^{(w)}) + l_x^{(r)},$$

so that $q_x^{(T)}$ is a weighted average of $q_x^{(h)}$ and $q_x^{(r)}$. Since $q_x^{(r)} \geq q_x^{(T)}$ is an empirically obvious inequality, we have $q_x^{(h)} \leq q_x^{(T)}$.

(b) For all x , we have ${}_t p_x^{(h)} = \prod_{k=0}^{t-1} p_{x+k}^{(h)}$. Similarly, ${}_t p_x^{(T)} = \prod_{k=0}^{t-1} p_{x+k}^{(T)}$. From Proposition 1 (a), $q_x^{(h)} \leq q_x^{(T)}$ so that $p_{x+k}^{(h)} = 1 - q_{x+k}^{(h)} - r_{x+k} - q_{x+k}^{(w)} \geq 1 - q_{x+k}^{(T)} - r_{x+k} - q_{x+k}^{(w)} = p_{x+k}^{(T)}$ for all $x > 0$ and for all k . It follows that ${}_t p_x^{(h)} = \prod_{k=0}^{t-1} p_{x+k}^{(h)} \geq \prod_{k=0}^{t-1} p_{x+k}^{(T)} = {}_t p_x^{(T)}$.
■

Theorem 1 *The industry model underprices the LTC Rider Cost.*

Proof.

Let

$$Ben'_{x+t} = \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} FA.$$

By substituting Ben'_{x+t} in (5), we have

$$C_{(rc)} = \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} (Ben_{x+t} - Ben'_{x+t}).$$

Likewise in (2), we have

$$C_{(ri)} = \sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} (Ben_{x+t} - Ben'_{x+t}).$$

We show that $(Ben_{x+t} - Ben'_{x+t}) \geq 0$. If death occurs at $x + t + u$ for a life impaired at $x + t$, then

$$\begin{aligned} Ben'_{x+t} &= v^{t+1+u} FA \\ Ben_{x+t} &= B_{x+t+1} v^{t+1} + \cdots + B_{x+t+u+1} v^{t+u+1} \\ &\quad + [FA - (B_{x+t+1} + \cdots + B_{x+t+u+1})] v^{t+u+1} \end{aligned}$$

Collecting coefficients, we have:

$$Ben_{x+t} = v^{t+1+u}FA + B_{x+t+1}(v^{t+1} - v^{t+u+1}) + \dots + B_{x+t+u}(v^{t+u} - v^{t+u+1})$$

Since $(v^{t+u} - v^{t+u+1}) > 0$ then

$$v^{t+1+u}FA + B_{x+t+1}(v^{t+1} - v^{t+u+1}) + \dots + B_{x+t+u}(v^{t+u} - v^{t+u+1}) \geq v^{t+1+u}FA$$

$$Ben_{x+t} \geq Ben'_{x+t}.$$

Now,

$$C_{(rc)} - C_{(ri)} = \sum_{t=0}^{\omega-x} [{}_t p_x^{(h)} - {}_t p_x^{(T)}] r_{x+t} (Ben_{x+t} - Ben'_{x+t}).$$

Since $({}_t p_x^{(h)} - {}_t p_x^{(T)}) \geq 0$ for all t, x from Proposition 1 (b), it follows that $C_{(rc)} - C_{(ri)} \geq 0$. ■

Theorem 2 *The industry model underprices the Stand-Alone LTC Benefit.*

Proof. Using Proposition 1 (b), we have

$$\sum_{t=0}^{\omega-x} {}_t p_x^{(T)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} B_{x+t+u+1} \leq \sum_{t=0}^{\omega-x} {}_t p_x^{(h)} r_{x+t} \sum_{u=0}^{\omega-x-t} {}_u p_{x+t}^{(r)} q_{x+t+u}^{(r)} v^{t+1+u} B_{x+t+u+1}, \text{ or } C_{(si)} \leq C_{(sc)}. \quad \blacksquare$$

3.3 Numerical Illustrations

It is of interest to determine the magnitude of the underpricing by the industry model. The results in Table 1 below are obtained under the following assumptions. (i) age of insured: 60; (ii) unreduced Death Benefit: 500,000; (iii) monthly LTC benefit: 10,000; (iv) Maximum LTC Benefit: 500,000; (v) waiting period: 90 days; (vi) interest rate: 7.75% compounded monthly; (vii) mortality rates : 75-80 basic tables for non-impaired lives, 3 times the 75-80 basic tables for impaired lives; (viii) incidence rates: (TSA Report, 1995) (ix) constant force of mortality assumption; (x) withdrawal occurs at the beginning of the month and before disablement; (xi) disablement occurs at the beginning of the month; (xii) death occurs at the end of month; (xiii) lapse

rates: 10% for the first year, 8% for the second year and 5% thereafter; and (xiv) LTC benefits are paid at the end of the month as is the death benefit.

Table 1 shows that the rider cost is underpriced by 1.04% by the industry model and the stand-alone cost by 1.72%. Table 2 shows how the error is impacted by the issue age of the insured. There is really no discernible pattern in the variation of the error from age to age except that the error in the stand alone cost is bigger than in the rider cost. Table 3 shows the effect on the error of changing the multiple of the base mortality rate to derive the impaired mortality rate. As in Table 2, the error in stand alone cost is bigger than in the rider cost. In addition, the error decreases as the factor increases, both in the rider cost and in the stand alone cost, except that a jump is noticeable at the factor 4 for the stand-alone.

4 Discussion

The deterministic model presents two shortcomings:

- With deterministic pricing, there is a failure to consider volatility of the loss random variable at issue, where the loss involved is the excess of the present value of benefits paid (and any remaining death benefit for the LTC Rider) over the present value of the unreduced death benefit.
- Standard industry LTC pricing generally assumes two sets of mortality rates: one for the overall population, and one for lives receiving LTC benefits, usually expressed as a multiple of the overall population mortality. The industry model fails to explicitly model the mortality of healthy lives.

These issues can be overcome by using a stochastic pricing model, which will be discussed elsewhere.

References

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Table 1: Industry Model vs. Theoretically Correct Model for Age 60

Model	Rider Cost nsp	Stand-Alone nsp
Industry	6,211.05	12,385.93
Correct	6,275.76	12,599.25
Error	1.04%	1.72%

Table 2: Impact of age of insured on error

Age	Rider Cost Error	Stand Alone Cost Error
60	1.04%	1.72%
62	1.68%	3.20%
64	0.19%	2.83%
66	0.21%	1.23%
68	1.32%	1.34%
70	2.76%	2.93%

Table 3: Impact of disabled life mortality factor on error

Age	Rider Cost	Stand Alone Cost
$2q_x^{(T)}$	2.42%	2.92%
$2.5q_x^{(T)}$	2.08%	2.57%
$3q_x^{(T)}$	1.04%	1.72%
$3.5q_x^{(T)}$	0.49%	1.06%
$4q_x^{(T)}$	0.46%	2.08%