Problem 1. Suppose \( \{X_t\} \) is generated by a stationary AR(3) process with zero mean, i.e., \( X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + Z_t \), where \( Z_t \sim WN(0, \sigma^2) \).
Let \( X_1, \cdots, X_n \) denote \( n \) observations from this process.

(a) Write down the general difference equation form of the \( l \)-step ahead MMSE prediction.

(b) Compute the one-step-ahead MMSE prediction \( X_n(1) \) and the two-step-ahead MMSE prediction \( X_n(2) \) in terms of the observed data.

(c) What are the distributions of the corresponding forecast errors \( e_n(1) \) and \( e_n(2) \)?

(d) Using the above information, describe how you will construct \( 100(1 - \alpha)\% \) prediction intervals for \( X_{n+1} \) and \( X_{n+2} \) (show the steps for numerical evaluation - simplified algebraic forms are not essential).

Problem 2. Suppose \( X_t = Z_t + \theta Z_{t-1} \), where \( |\theta| < 1 \) and \( Z_t \sim WN(0, \sigma^2) \).
Compute the one-step-ahead MMSE prediction \( X_n(1) \) in terms of \( \{X_t, -\infty < t \leq n\} \) (you may use the \( \pi \) weights of the process).

Problem 3. Identify suitable ARIMA models for the following data described in B & D (you may use PROC ARIMA in SAS). Note that in some cases, it might be useful to fit an ARIMA model to some transformation of the raw data (say, a log transform). In each case, it is possible you identify a set of possible candidate models; justify why you choose these based on patterns of the sample ACF and sample PACF.

a) Data is in uspop.xls

b) Data is in accdeaths.xls

c) Data is in airpass.xls

Extra Problems - Mandatory for ST380 students

1*. Brockwell and Davis, Exercise 6.6

2*. Brockwell and Davis, Exercise 6.11